Exercise 35

Patients undergo dialysis treatment to remove urea from their blood when their kidneys are not functioning properly. Blood is diverted from the patient through a machine that filters out urea. Under certain conditions, the duration of dialysis required, given that the initial urea concentration is $c > 1$, is given by the equation

$$t = \ln\left(\frac{3c + \sqrt{9c^2 - 8c}}{2}\right)$$

Calculate the derivative of t with respect to c and interpret it.

Solution

Take the derivative of t with respect to c by using the chain rule repeatedly.

$$\begin{aligned}
\frac{dt}{dc} &= \frac{d}{dc}\ln\left(\frac{3c + \sqrt{9c^2 - 8c}}{2}\right) \\
&= \frac{1}{\frac{3c+\sqrt{9c^2-8c}}{2}} \cdot \frac{d}{dc}\left(\frac{3c + \sqrt{9c^2 - 8c}}{2}\right) \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \frac{d}{dc}\left(3c + \sqrt{9c^2 - 8c}\right) \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \left(3 + \frac{d}{dc}\sqrt{9c^2 - 8c}\right) \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \left[3 + \frac{1}{2}(9c^2 - 8c)^{-1/2} \cdot \frac{d}{dc}(9c^2 - 8c)\right] \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \left[3 + \frac{1}{2}(9c^2 - 8c)^{-1/2} \cdot (18c - 8)\right] \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \left(3 + \frac{9c - 4}{\sqrt{9c^2 - 8c}}\right) \\
&= \frac{1}{3c + \sqrt{9c^2 - 8c}} \cdot \left(\frac{3\sqrt{9c^2 - 8c} + 9c - 4}{\sqrt{9c^2 - 8c}}\right) \\
&= \frac{3\sqrt{9c^2 - 8c} + 9c - 4}{\left(3c + \sqrt{9c^2 - 8c}\right)\sqrt{9c^2 - 8c}}
\end{aligned}$$

This represents the rate that the duration of dialysis increases as the initial urea concentration increases.

Below is a graph of t and its derivative versus c.